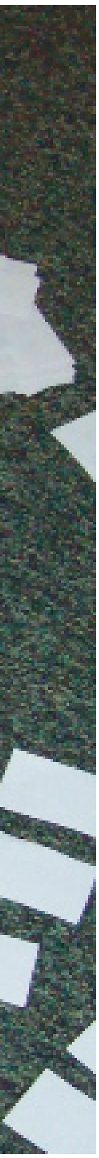
Fluency with Basic Addition

Applying known facts to derive unknown facts results in efficiency, flexibility, and an understanding of number combinations for young students.

By Gina Kling



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What should our goals be when it comes to having our students learn basic addition facts? The national Common Core State Standards (CCSSI 2010a) and other state standards have expectations that refer to *fluency*. For example, first graders must demonstrate "fluency for addition and subtraction within 10," and second graders must "fluently add and subtract within 20 using mental strategies" (CCSSI 2010a). Similarly, *Principles and Standards for School Mathematics* affirms the expectation that students in the early elementary grades will "develop fluency with basic number combinations for addition and subtraction" (NCTM 2000, p. 78). But what exactly does fluency mean, and how might fluency differ from having instant recall of each and every basic fact? The goal of this article is to examine what it means to be fluent with basic addition facts and to focus on activities that teachers can use to prepare students for fluency. Let's begin by defining what is meant by *fluency with basic addition facts*, then focus on strategies that fluent students use, and finally step back to look carefully at several activities that can prepare students to learn core facts needed to become fluent.

What fluency means

Traditionally, learning basic facts has focused on rote memorization of isolated facts, typically through the use of flash cards, repeated drilling, and timed testing. However, as many experienced teachers have seen, "drill alone does not develop mastery of single-digit combinations" (Kilpatrick, Swafford, and Findell 2001, p. 192). In contrast, a fluency approach to learning basic addition facts places a focus on developing and using mathematical strategies, with the goal of finding efficient, effective ways to apply *known* facts to *derive* unknown facts. For example, if a student did not know the answer to 7 + 5 (often a difficult fact for first graders), she could simply think of adding 5 + 5 and then add 2 more. Here the student uses a fact she is likely to know (5 + 5) to derive an unknown fact in an efficient, meaningful way. Thus, students who struggle to learn facts or who often forget certain facts have an alternative approach to fall back on that allows for more complex mathematical thinking than simply counting. Furthermore, learning to decompose and recompose numbers in flexible ways is an important step in students' development of efficient computational strategies (Wheatley and Reynolds 1999), and this type of thinking transfers to multidigit computation (such as applying the known-facts strategy above to solve 50 + 70).

A common misconception of the fluency approach is that neither speed nor memorization is important, which could not be further from the truth. Being efficient and memorizing some facts (that can be used to derive unknown facts) are essential components of fluency. However, note the difference between speed and efficiency: A student who can instantly recall a fact is obviously doing so with a great deal of speed, but a student who needs a few seconds to mentally work through a meaningful strategy to derive a fact is certainly more efficient (and employing better mathematical thinking) than a student who must resort to counting to figure out the unknown fact. Furthermore, from an assessment standpoint, simply because a student is (or is not) capable of quickly recalling facts indicates nothing about true understanding of the mathematics involved. Students with deep mathematical understanding might still struggle to instantly recall a fact, particularly in a pressurized situation like a timed test. Students who can quickly recall facts might be good at memorizing but may be quite immature in their mathematical understanding and thinking:

> [The] speedy recollection of facts should not be confused with real mathematical skill. Good mathematical strategies—not quick memorization—are what really matter in understanding mathematics. (Mokros, Russell, and Economopoulos 1995, p. 72)

In summary, fluency with basic addition facts can be defined as "the efficient, appropriate, and flexible application of single-digit calculation skills and is an essential aspect of mathematical proficiency" (Baroody 2006, p. 22). Fluent students use the facts they have memorized in flexible, mathematically rich, and efficient ways to derive facts they do not know. Finally, fluent students are able to demonstrate effective thinking strategies that involve decomposition and recomposition of numbers and, as such, have had opportunities to develop more advanced mathematical understanding than their counterparts who have been limited to rote memorization and drilling of their basic facts. But which strategies are these fluent students likely to use, and how can their teachers help them develop those strategies? We now turn our attention to those two ideas.

Strategies fluent students use

If fluency is the goal, classroom instruction must emphasize the development and application of strategies; rote memorization of isolated facts will not suffice to develop proficiency with basic facts (Kilpatrick, Swafford, and Findell 2001). One way to do this is by simply taking ordinary flash cards and changing them into clue cards. This activity is used in the NCTM Standards-based Investigations in Number, Data, and Space curriculum but is widely adaptable to fit any curriculum. To begin, students could sort addition fact cards into piles of those they know and those they do not instantly know. For facts that are not automatic, they then record a clue onto each card that they could use to help them derive that fact. So, for the example of 7 + 5, a student might record the clue 5 + 5 on his card. In essence, students learn to recognize a starting point to work from for each fact that they need to practice, a powerful alternative to counting.

TABLE 1

When students master two types of facts—doubles and combinations that make 10—they can derive nearly every other challenging fact on this table.

+	0	1	2	3	4	5	6	7	8	9
0	0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0
1	0 + 1	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1	9 + 1
2	0 + 2	1 + 2	2 + 2	3 + 2	4 + 2	5 + 2	6 + 2	7 + 2	8 + 2	9 + 2
3	0 + 3	1 + 3	2 + 3	3 + 3	4 + 3	5 + 3	6 + 3	7 + 3	8 + 3	9 + 3
4	0 + 4	1 + 4	2 + 4	3 + 4	4 + 4	5 + 4	6 + 4	7 + 4	8 + 4	9 + 4
5	0 + 5	1 + 5	2 + 5	3 + 5	4 + 5	5 + 5	6 + 5	7 + 5	8 + 5	9 + 5
6	0 + 6	1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6	7 + 6	8 + 6	9 + 6
7	0 + 7	1 + 7	2 + 7	3 + 7	4 + 7	5 + 7	6 + 7	7 + 7	8 + 7	9 + 7
8	0 + 8	1 + 8	2 + 8	3 + 8	4 + 8	5 + 8	6 + 8	7 + 8	8 + 8	9 + 8
9	0 + 9	1 + 9	2 + 9	3 + 9	4 + 9	5 + 9	6 + 9	7 + 9	8 + 9	9 + 9

Key to strategies (Only one strategy per combination is identified here, although many facts could be found using multiple strategies.)

	"Duplicates" if students understand the commutative property		Near doubles (doubles ±1 or 2)
	Relates to the meaning of addition or counting sequence		Combinations of 10
	Pairs that are near 10 or easy to count on or back to 10		Doubles

When working with elementary school teachers on this idea, I often have them sort different fact cards into piles on the basis of strategies they would use to solve them. An analysis of different sorting schemes allows several striking patterns to emerge, helping us recognize crucial strategies for mastering basic facts. I highlight these strategies in our discussion, using **table 1** to help illustrate these ideas.

By focusing on patterns that arise from the clue card sort, we begin to greatly simplify the task of mastering single-digit addition combinations. First of all, if students develop an understanding of the *commutative* property (i.e., the order of the addends is reversible in a problem dealing strictly with addition), then we can cut **table 1** almost in half to begin with. Next, we discuss how we do not need to memorize facts at all when an addend is 0. Simply by understanding the meaning of adding 0, children quickly learn these facts. Furthermore, facts with an addend of 1 or 2 are quite simple for students because they relate so closely to the counting sequence, and thus are easily memorized.

We now turn our attention to specific ideas that arise in the clue card sort. For example, we always have a pile of doubles—that is, such facts as 3 + 3, 4 + 4, and so on—followed by facts that are near doubles—that is, 1 or 2 away from a double (see **table 1**). Research has shown that students quickly memorize their doubles (Kilpatrick, Swafford, and Findell 2001), and if students have their doubles memorized, they can simply derive any of the near-double facts and thus do not need to commit those to memory. For example, if a student does not know the sum of 6 + 7, he could simply use the known double of 6 + 6 and then add 1 to the sum. Also identified in the clue card sort are the combinations of 10 (3 + 7, 4 + 6, etc.), the learning of which is a heavy emphasis in many other countries (Kilpatrick, Swafford, and Findell 2001), and for good reason. Students who have memorized this set of facts can then apply a make-10 strategy when faced with such facts as 4 + 7 (3 + 7 and then add 1). Similarly, all the facts that have an addend of 9 can be handled by first making 10 and then adding what remains, a simpler computation because adding to a 10 is generally easier for students.

Of course, with any of these combinations, students may think in multiple ways. For example, given such a fact as 9 + 6, one student may think, I'll pretend the 9 is a 10, and I know that 10 + 6 gives me 16. But now, since I added 1 too many when I made 9 into 10, I'll take 1 away to get 15.

Another student may decompose 6 directly into 1 and 5 and then combine 1 with 9 to make 10 before finally adding 5 (in essence, using the decomposition of numbers and the associative property of addition). Clearly, when the focus is on fluency instead of just rote memorization, much potential exists for sharing and discussing good mathematical thinking in the classroom. This practice is an important technique for helping students make sense of and retain these important strategies, leading to increased success (Kilpatrick, Swafford, and Findell 2001). For more information on how to develop all these strategies in your classroom, a terrific resource is the article "The Road to Fluency and the License to Think" (Buchholz 2004).

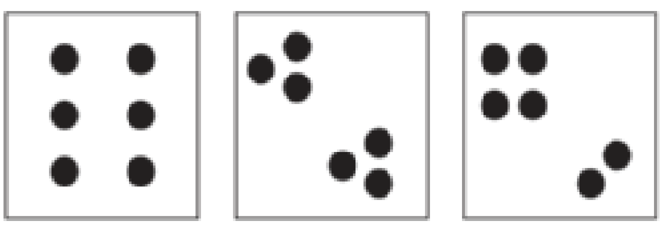
On careful study of **table 1**, one begins to recognize that if students can master two types of facts—the doubles and the combinations that make 10—they can then derive nearly every other challenging fact on the table. This is an empowering concept for teachers and their young students, offering a road map for how classroom instruction on basic facts might proceed. However, this understanding brings with it the temptation to resort to rote memorization of these sets of facts. In contrast, to help students memorize these facts in a *meaningful* way, we must seek out specific, motivating experiences for them to engage in. The rest of this article outlines several engaging activities, some of which can begin as early as preschool, that will allow students to acquire these most critical basic facts through experience.

Preparing young students

For several years I have been sharing with both preservice elementary school teachers and practicing preschool through grade 2 teachers a number of different activities designed to help prepare their students for fluency with basic addition facts. The first two types of activi-

FIGURE 1

Regular practice with dot patterns allows students to recognize quantities without counting and to recognize single-digit addition combinations.



ties—ten-frames and quick images with dot patterns—can be used as early as preschool to help students learn to think of numbers in flexible ways. And for slightly older students, several games from the Investigations curriculum can help students learn their combinations of 10 through practice that is engaging and motivating.

The idea of a *quick image* is to quickly show a representation to students with the expectation that they will retain a mental picture of what they saw and then use that image in some way. This activity is easily implemented using an overhead projector. For example, quick images might be used in geometry by flashing a two-dimensional depiction of a multifix cube structure for a few seconds and then asking students to construct what they saw by using their mental image and actual multifix cubes. Or, for the purposes of developing number sense, we might flash a pattern of dots on the screen and ask students how many were shown and how they saw them. The key would be to flash the image quickly enough so that students could not rely on counting to determine their answer. Such an activity serves several purposes, namely, to move students away from counting and toward *subitizing* (instantly seeing the quantity), to help them recognize numbers as a collection of items that can be decomposed in different ways (Wheatley and Reynolds 1999), and to help them recognize different possible representations of a number. For example, in **figure 1** we see 3 possible dot patterns, each with a total of 6 dots. What might be gained from considering each? In the first, we recognize the common depiction of 6 that we see on number cubes, dominos, and so on. Adults easily identify 6 without any need to count. In the second example, we begin to recognize 6 as the double of 3, or that 3 + 3 = 6. And in the third example, we see 6 now decomposed into 4 plus 2. Regular practice with dot patterns allows students to start recognizing not only quantities without counting but also single-digit addition combinations, without even realizing they are learning their basic facts.

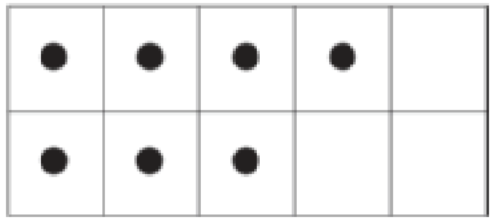
Another powerful application of the quick-image idea is to place counters into a ten-frame, or grid, containing 2 rows of 5 squares each. For example, a teacher may flash a ten-frame (see **fig. 2**) on the overhead for two seconds and ask her students which number it represents and how they recognized it. Some students will have thought of 7 as 4 + 3; others might have seen it as 6 + 1. And still others may have noted 3 empty squares, used their knowledge of the structure of a ten-frame as having 10 squares, and then computed 10 – 3 to find 7 counters present (known as the *complement*). Finally, students also may have thought of decomposing the 3 dots on the bottom into 2 and 1 and mentally moving 1 dot to the top row to make 5 + 2.

In each of these cases, we see essential practice with combinations of 7, practice that integrates mental imagery with the flexible decomposition and recomposition of numbers and that focuses on relating numbers to 5 and 10 due to the structure of a ten-frame. Of course, work with a ten-frame is not limited to the quick-image format (although quick images may be necessary to push students beyond counting). Another use is to ask students to represent different numbers or sums of numbers on a single or a double ten-frame (a double ten-frame is composed of two side-by-side ten-frames). For example, when introducing the ten-frame to teachers, I often ask them to find several ways of representing the number 8. (I encourage readers to take a moment to do this as well before proceeding.) Many possibilities emerge, including patterns that elicit the idea of 8 as a double of 4 (this can occur in more than one way) as well as 8 as the difference between 10 and 2. Thus the two key strategies—combinations of 10 and doubles—that were highlighted in the previous section are likely to be used often as students work with their ten-frames.

Dot patterns and ten-frames can be used as

FIGURE 2

Showing counters in a grid to students as young as preschool age is a powerful application of the quick-image idea.



ANDRA ZOMMERS

Classroom games help students develop fact fluency.

early as preschool (there are also five-frames, consisting of a single row of 5 squares, for preschoolers to start with) through the early elementary school grades. Such activities work well when done periodically over the course of the year, such as for a warm-up activity (Kline 1998), so that over time students have an opportunity to build their visualization and number pattern recognition skills. To further reinforce the development of combinations of 10, I have found that two games from the Investigations textbook *Coins, Coupons, and Combinations* (Economopoulos 1998) have worked well with students in the first and second grades. These games are Tens Go Fish and a modified version of Turn Over Ten, which both use a deck consisting of four sets of 0–10 number cards (or one could use regular playing cards, numbered 1–9). Tens Go Fish is played much like the traditional Go Fish game that most first graders are well-acquainted with, the key difference being that rather than looking for matching pairs, students look for pairs of numbers that add to 10. They begin by each drawing 5 cards and taking turns asking each other for a needed card. For example, if a student has a 3, she would ask a player of her choice if he has a 7. If that player indeed has a 7, he hands it over; otherwise he says, "Go fish," and the first player takes a new card from the deck. Play continues until no more cards are in the deck. Likewise, Turn Over Ten also focuses on combinations of 10 and can be played in a manner similar to the popular Memory game by having students set up their cards in a 4×5 array and turn over 2 cards, 1 at a time, with the hope of finding a pair that sums to 10. If such a pair is found, the cards are added to the player's own pile; if not, the cards are turned back over and all players try to remember the contents of the previously viewed cards to help in finding pairs that equal 10 in the future.

Turn Over Ten and Tens Go Fish both offer students the meaningful practice that is necessary for them to commit the combinations of 10 to memory, and both games have received positive feedback from the first- and second-grade teachers (as well as their students) with whom I have worked. One first-grade teacher reported to me that as they played these games, her students engaged in mathematical conversations with one another, making suggestions and asking questions. Justification became a natural outcome of playing the games as the students attempted to convince their classmates that they had indeed made a sum of 10. This teacher felt that her students were much more engaged and employed deeper mathematical thinking than when playing the typical games

MARSHA GARZA

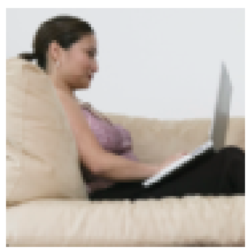
A regular card deck and such games as Tens Go Fish make learning number combinations easy and fun.

for their mathematics curriculum. According to that teacher, "I used these activities often, and they became the most requested games in my first-grade classroom."

Furthermore, a grade 2 teacher who used these games near the end of the school year commented that they helped her students refocus on the importance of working with the number 10. After seeing how helpful they were, this particular teacher contacted me for more activities like them. See the **online appendix** for additional basic-facts games I shared with her. Other elementary school teachers have confirmed the success of such activities as ten-frames, Turn Over Ten, and Tens Go Fish as well (Barker 2009).

Conclusion

Activities like quick images with dot patterns, ten-frames, and games—such as Tens Go Fish and Turn Over Ten—are not only engaging for students and helpful for memorizing key facts, doubles, and combinations of 10 but are also grounded in essential mathematical ideas. When students participate in these activities, they actively construct and manipulate images of numbers, decomposing and recomposing those numbers to create combinations that are easier for them to work with. As they do so, they begin to recognize sums and differences of single-digit numbers and to use known number patterns and combinations to help determine unknown ones. In essence, they begin to develop fluency with their basic addition facts without even realizing it. Teachers of such students can then work from these rich experiences—which so often focus on



MARSHA GARZA

Math games lead to rich mathematical conversations as students ask questions and justify their thinking.

combinations of 10 and doubles—to cultivate the development of further strategies, such as near doubles and make 10, with their students. Much as fluent speakers of a language need not know every word of the language, these students may not have every addition fact committed to memory, but they have enough facts memorized that they can quickly, effortlessly derive any unknown fact. These students have achieved fluency with their basic addition facts and are well on their way to becoming confident, creative, and strategic users of mathematics, ready to tackle the complexities of the math that lies beyond the addition table.

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Go to **www.nctm.org/tcm** to access the **appendix**, which accompanies the online version of this article.